Practical notes

1 Metamorphic time and the significance of textures

One of the primary aims of the metamorphic geologist is to decipher the *information content* of metamorphic rocks. The relevant information provides a kind of *travelogue* relating to the rocks burial into, and excavation out of, the metamorphic pile. The relevance of metamorphic geology is that this travelogue provides one of the most important datasets constraining the evolution of ancient orogenic belts. This travelogue is preserved in the language of mineral associations, fabrics and textural relations (as well as major element and isotopic compositions) of the metamorphic rocks preserved in the orogenic belt. In particular, the relevant information pertains to

- the sequence of pressure and temperature conditions encountered by the rocks during their metamorphic journey, which is reflected in the mineral associations and the textural relations pertaining to reactions in individual rocks (note that the way the PT history of individual rocks varies across an orogenic belt also provides critical information)

- the history of deformation, which is reflected in the fabrics and microstructures, and

- the interaction with fluids (including both H_2O-CO_2 fluids and silicate melts) which may have infiltrated into, or been liberated from, the metamorphic rock; the record of which is reflected in the gross chemical character of the rock.

It should be noted that many ancient rocks have a complex history involving repeated metamorphism (polymetamorphism), the record of which preserves a *memory* of more than one orogenic event. Reading this record without ambiguity or confusion is by no means trivial.

In all of this, the use of textural relations to establish relative timing of the growth and dissolution of the various phases that currently make up, and may once have formed part of, an

individual rock is of the utmost importance. More than anything else the textures provide us with the clue to the direction of *metamorphic time*.

Examine the thin sections provided with a view to establishing the relative growth and dissolution history of the main phases. The idea is to appreciate the direction of metamorphic time in terms of the changing mineral composition of the rocks. Additionally, make sure you know the mineral chemistry and optical properties of all the phases, which you were expected to have learned in second year but have no doubt forgotten. For two thin sections make a sketch showing the important textural relations amongst the minerals and concisely (one paragraph!) describe the evidence for the passage of 'metamorphic time'.

Question 1.1

Some terms that will aid your descriptions include (you may need to refer to the second year notes for a refresher):

- symplectite

- corona

- porphyroblast

- poikiloblast

- inclusion trail

- foliation

- 'pressure' shadow

- micro-boudinage

- pseudomorph

- mylonite

Be Warned! **You may be given random tests on the composition and distinguishing features of any minerals that you have seen in the pracs - so make an effort to learn the compositions of the minerals as you come across them.**

2 Pressure and temperature

These preliminary exercises are designed to focus our minds on the basic physics governing pressure and temperature distributions in the Earth. The quantitative evaluation of physical relationships requires attention to both the *magnitude* and the *dimensions* of the various parameters, remembering that a dimensionally valid expression requires that all units are balanced! Frequently, important insights into the factors controlling the behaviour of physical systems can be obtained simply by considering the dimensions of the various parameters without regard to magnitudes. This is particularly evident when we try and understand the question *how long does it take to cook a rock?* Consequently, the following notes concentrate on the dimensions of the various parameters governing pressure and temperature distributions in the Earth. You will be expected to be familiar with the various units!

Pressure

The vertical stress at any point in the Earth is given by the *weight* of overlying rock, and is obtained by integrating the product of the gravitational acceleration (m s^{-2}) with the mass density (kg m^{-3}) from the surface to the depth of interest:

$$\sigma_{zz} = \int_0^z g\,\rho\,dz,$$

Any definite integral of this type simply represents the area under a curve of the *intergrand*, $(g\,\rho)$, plotted as a function of the *variable of integration*, z.

For homogeneous mass density distributions (that is, where the density does not vary with depth) this integral expression is just :

$$\sigma_{zz} = g\,\rho\,z,$$

Stress is measured in Pascals (Pa), the SI units being kg m^{-1} s^{-2}. In geology the most convenient units of vertical stress are MPa ($= 10^6$ Pa). Note that in much of the geological literature pressures are referred to in kilobars (kbars) with 1 kbar equivalent to 100 MPa.

Stress is defined as *force divided by area* in the limit that the area vanishes, and force is given by *mass times acceleration*

Calculate the vertical stress at the base of a crust consisting of an upper, 12 km thick layer of granite with density 2650 kg m^{-3} and a lower, 18 km thick layer of mafic granulite with a density of 2900 kg m^{-3}. Does the pressure at the base change if the order of the layers reversed? You may also consider the somewhat

Question 2.1

more challenging question of whether the potential energy of the system changes if the layers are reversed.
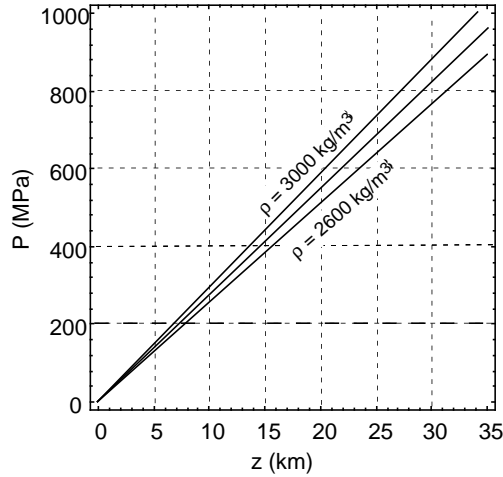

Figure 1: Correlation between pressure (in MPa) and depth (km) for typical crustal densities.

Metamorphic geologists usually equate pressure with the vertical stress (Figure 1) and therefore changes in the pressures recorded by metamorphic rocks are usually equated with changes in depth. More strictly, pressure is given by the mean stress, defined in terms of the three principal stresses at a point:

$$P = \frac{1}{3}\left(\sigma_1 + \sigma_2 + \sigma_3\right)$$

The vertical stress is usually one of the principal stresses, with $\sigma_{zz} = \sigma_1$ for normal fault stress regimes, $\sigma_{zz} = \sigma_2$ for strike-slip fault stress regimes and $\sigma_{zz} = \sigma_3$ for reverse fault stress regimes.

In a reverse-fault, plane-stress regime the difference between the mean stress (or pressure) and the vertical stress is given by the deviatoric or shear stress, τ (note that it is this shear stress that gives rise to deformation of rocks). The magnitude of shear stresses associated with deformation in orogenic belts (ie., during metamorphism) is the subject of ongoing debate. Some workers think that the crust is weak ($\tau < 20$ MPa), largely because there is little direct evidence for the sort of frictional heating that maybe expected along major faults such as the San Andreas Fault, if the crust were strong. Others think the crust is relatively strong and is able

A plane stress regime is one where the intermediate stress, σ_2, is the same as the mean stress

to support shear stresses of up to about 100 MPa, based on apparent *inverted metamorphic gradients* associated with deeply exhumed thrusts such as the Main Central Thrust in the Himalaya (curently I am a *fence-sitter* in this debate, albeit one who is intrigued by the possibile ramifications of high shear strengths. Geoff Fraser is working on this very problem - so you can ask him for the latest on the Main Central Thrust argument!)
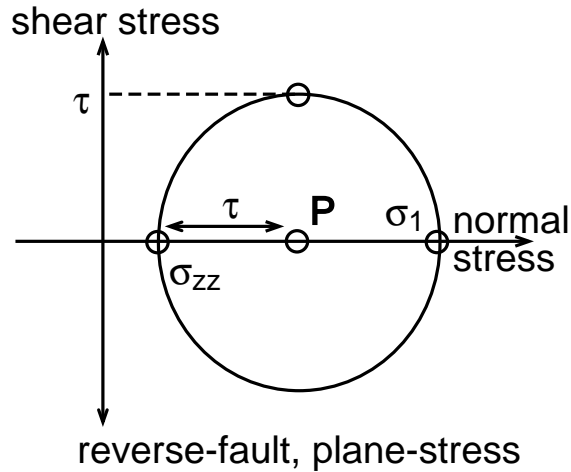

Figure 2: Mohr diagram construction showing the relationship between pressure, P, vertical stress, σ_{zz}, and shear stress, τ, in a reverse-fault, plane-stress environment.

Thermal units

The temperature distribution in the Earth reflects its thermal properties such as heat capacity, heat production and thermal conductivity. The units appropriate to these properties are described below.

Heat Capacity and Content. The heat capacity, c_p, of a material is the measure of the amount of energy needed to raise the temperature of a unit mass by a unit degree and therefore is measured in units of J kg^{-1} K^{-1}. The heat content of the material, per unit mass (ie., J kg^{-1}), is given by the product of heat capacity and temperature ($C_p T$), or, per unit volume (ie., J m^{-3}), by the product of heat capacity, density and temperature ($C_p \rho T$).

Heat Production and Heat Flow. The natural units of heat production and heat flow must incorporate a time dimension appropriate to its production and transport. Thus, for heat production per unit mass, H, the natural units are J s^{-1} kg^{-1} or more ususally W kg^{-1} where the *Watt* has units of J s^{-1}. Often heat production is referred to per unit volume (W m^{-3}) which is readily obtained by multiplying the heat production per unit mass (W kg^{-1}) by the density (kg m^{-3}). Typical heat production rates for crustal rocks are of the order of a few times 10^{-6} W m^{-3} or a few μW m^{-3}. Granites typically have elevated heat production rates, with extremely radioactive granites having heat production rates greater than about 10 μW m^{-3}. The Mount Painter region in the Northern Flinders Ranges is exceptionally 'hot', with some granitic gneisses in this region having heat production rates as high as 40 μW m^{-3}.

Heat flow is measured in units of energy passing through a unit area in a unit time. Thus, the natural units of heat flow are W m^{-2} (or J s^{-1} m^{-2}). The magnitude of typical surface heat flows on the continents is of the order of 0.06 - 0.08 Wm^{-2} or 60 - 80 mW m^{-2}. Archaean cratons, such as the Yilgarn, tend to have low heat flows (about 40-50 mWm^{-2}), while tectonially active regions such as the Basin and Range province in the western US tend to have much higher heat flows (greater than about 80 mWm^{-2}). For its age, the Australian continent seems to be anomalously hot, particularly the Proterozoic provinces of South Australia, the Northern Territory and western Queensland, where the average heat flow averages about 85 mWm^{-2}. The unusually high heat flows may be responsible for many of the unusual geological features of this region and provides the motivation for Sandra Mclaren's PhD research.

Question 2.2

Use a dimensional analysis to determine how heat flow, q, relates to (1) volumetric heat production, $H\rho$, and (2) thermal conductivity, k. What factors might control variations in heat flow in different crustal terranes!

Thermal Conductivity and Diffusivity. The rate at which heat is transported by conduction, ie., the heat flow, is

dependent not only on the temperature gradient (with units of K m^{-1}) but also on a material dependent parameter known as the thermal conductivity, k. From Fouriers' law, $q = -k \operatorname{grad} T$, the natural units of thermal conductivity are given by W m^{-1} K^{-1} (ie., conductivity equals heat flow divided by the temperature gradient).

In many formulations of the heat conduction equation the material parameter is expressed as a thermal diffusivity, κ, rather than conductivity, where the units of thermal diffusivity are m^2 s^{-1} (ie., thermal diffusivity equals thermal conductivity divided by the heat capacity per unit volume).

Use a dimensional analysis to establish the relationship between κ, k, c_p and ρ.

Effect of thermal conductivity on temperature gradients

A steady-state is defined when there is no change in thermal energy content, and hence temperature, with time. However, heat can still flow down temperature gradients in the steady state with the proviso that the heat flowing out of a given material volume is exactly equivalent to the sum of the heat flowing into the volume and the heat produced in the volume. Note that this is simply an expression of the fundamental and most basic of physical *conservation principles*.

The conservation principle has tremendously wide-ranging application. Amongst other things it governs the state of your bank balance! Show how it might apply to the evolution of topography on a hillslope!

We begin by considering heat flow in the absence of internal heat production. According to Fourier's law heat flow is given by :

$$q = k \operatorname{grad} T$$

Thus, variations in conductivity should be expected to correspond to variations in temperature gradients. In the steady-state, in the absence of heat production, heat flow is constant, and consequently knowledge of variations in conductivity can be translated to variations in temperature gradients.

The thermal conductivities of rocks varies with mineralogy (Figure 4) as well as texture, and temperature. Quartzites
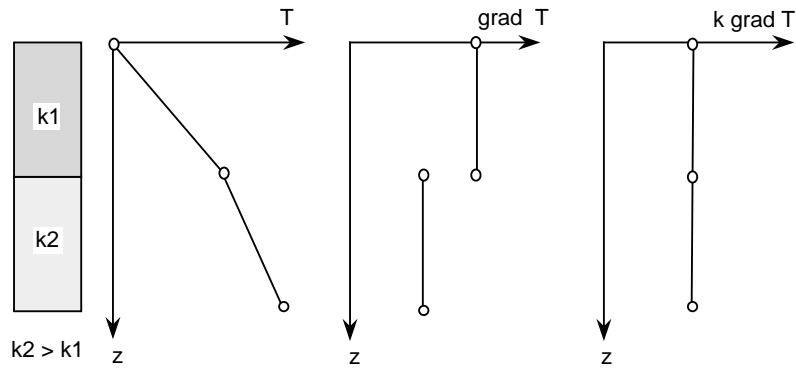
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Figure 3: Schematic illustration of relationship between T, grad T and k grad T (= q) for heat flow in composite material consiting of two layers,1 and 2, with conductivity of layer2 (k2) greater than the conductivty of layer 1 (K1).

typically have high thermal conductivity, while shales have low thermal conductivity.

Calculate the temperature-depth distribution in a horizontally layered quartzite-shale sequence in which a 2 km thick quartzite with conductivity of 6.2 W m^{-1} K^{-1} sits above a 3 km thick shale unit with a conductivity of 2.3 W m^{-1} K^{-1}. The measured surface heat flow is 75 mW m^{-2}. Assume that the top of the sequence is at the surface of the Earth, and choose your preferred geographic location. Does the temperature at the base of the sequence depend on the order of the layers?
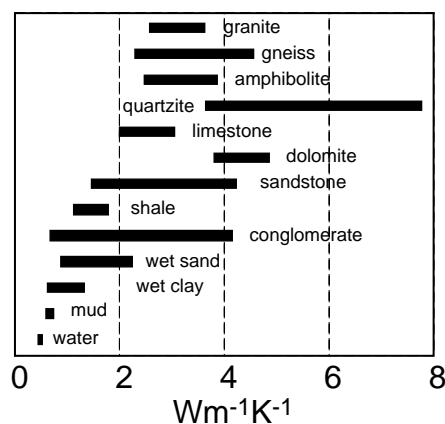
Question 2.4

Figure 4: The thermal conductivity ranges of some common rock types.

2.1 Steady-state Geotherms

Analytical expressions for geotherms (ie., temperature as a function of depth) can only be derived from the steady-state thermal-energy balance, providing we have a simple, continuous, analytical expression for the way heat production varies with depth. In reality, heat production distributions are likely to be complex, discontinuous functions of depth (mainly because geological formations have discrete boundaries). However, useful geotherms can be derived by assuming that the first-order form of the heat production distribution is some relatively simple function of depth. Two commonly used functional relationships are :

- a heat production distribution that is constant with depth ie., $H(z) = H_s$ where H_s is the heat production at the surface, and

- a heat production distribution that shows an inverse exponential dependance on depth ie., $H(z) = H_s \exp\left(-z/h_r\right)$, where h_r is a characteristic length scale for the distribution $(H(h_r) = H_s/e)$.
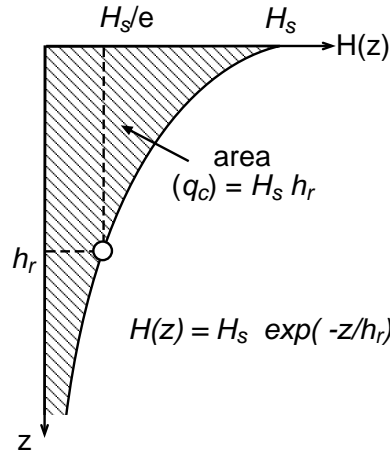

Figure 5: The exponential heat production model.

For heat production which reduces exponentially with depth the crustal contribution, q_c, to surface heat flow , q_s, is given by

$$q_c = \rho \, H_s \, h_r$$

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with the total surface heat flow

$$q_s = q_m + q_c = q_m + \rho \, H_s \, h_r$$

where q_m is the heat flow from the deeper mantle supplied to the base of the heat producing layer (ie., the base of the crust).

What is the expression relating surface heat flow, q_s, and surface heat production, H_s, for the constant depth distribution (assume that the heat producing layer is of thickness z_c).

Question 2.5

Use the data in Figure 6 to determine q_m and h_r for the western U.S.A.

Question 2.6

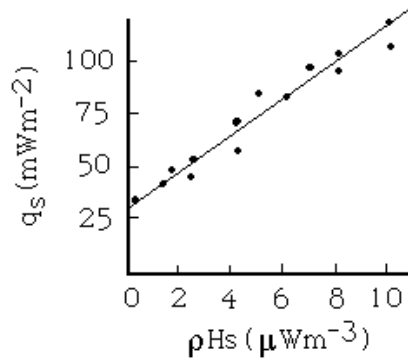Figure 6: Heat flow versus surface heat production in the western U.S.A (after Lachenbruch, 1967).

The temperature distribution at any depth, z, in terms of the mantle heat flow, q_m, the surface temperature, T_s, and the crustal thickness, z_c, for a homogeneously distributed heat source is given by

$$T_z = \frac{\rho \, H_s \, z \left(z_c - \frac{z}{2}\right)}{k} + \frac{q_m \, z}{k} + T_s \qquad (1)$$

Using the values: $T_s = 0^\circ$C, $z_c = 35$ km, $\rho \, H_s = 1.0 \, \mu$W m^{-3}, $k = 3$ W m $^\circ$K^{-1}, and $q_m = 30$ mW m^{-2}, smclaren-calculate the Moho temperature, the surface heat flow and the thickness of the lithosphere, z_l, assuming a basal temperature $T_l = 1280^\circ$C. High-temperature thermal conductivities of rocks are poorly known, with uncertainites of the order of $\pm 33\%$. What effect does this uncertainty in thermal conductivity have on our knowledge of the steady-state Moho temperature.

Question 2.7

For an exponentially decreasing heat production the equivalent expression relating temperature and depth is

$$T_z = \rho \, H_s \, h_r^2 \left(1 - e^{(-z/h_r)}\right) + \frac{q_m \, z}{k} + T_s$$

Using this relation and the values : $T_s = 0^o\mathbf{C}$, $\rho \, H_s = 2.5 \, \mu\mathbf{W \, m^{-3}}$, $k = 3 \, \mathbf{W \, m \,}^o\mathbf{K^{-1}}$, $h_r = \mathbf{10 \, km}$, **and** $q_m = \mathbf{30 \, mW \, m^{-2}}$, **calculate the thickness of the lithosphere,** z_l, **assuming a basal temperature** $T_l = \mathbf{1280}^o\mathbf{C}$.　　　　**Question 2.8**

Application to central Australia

Heat flow in the vicinity of Alice Springs is about 65 mWm^{-2} (which is relatively low for the Australian Proterozoic). Bottom-hole temperatures measured from exploration wells in the Amadeus Basin immediately south of Alice Springs indicate average temperature gradients of about 25°C/km.

What is the average thermal conductivity of the sediments in the Amadeus Basin?　　　　**Question 2.9**

Seismic tomographic studies (SKIPPY) point to a very thick (around 250 km) lithosphere beneath central Australia suggesting very low mantle (or reduced) heat flows.

Assuming a modern day Moho temperature (at a depth of about 40 km) of about 500°C, and that the mantle thermal conductivity is the same as the sediments in the Amadeus Basin (admitedlty, a big assumption), calculate the mantle or reduced heat flow - (assume that the temperature at the base of the lithosphere is around 1300°C).　　**Question 2.10**

The remainder of the surface heat flow observed in central Australia must be contributed by heat production (U,Th, K decay) within the crust. In parts of central Australia such as the Anmatjira Ranges, granitic gneisses presently exposed at the surface have unusually high heat production rates of between 5-10 μWm^{-3} (granites typically have heat production in the range 3-5 μWm^{-3}).

What thickness of granitic gneiss producing heat at 8 μWm^{-3} is required to account for the present day central Australian heat flow?　　**Question 2.11**

Such enriched layers suggest that heat production is unusually concentrated (in a vertical sense) in the central Australian Proterozoic crust. Moving such an enriched layer up and down in the crust by processes such as erosion, deformation and sedimentation may significantly influence mid and deep-crustal thermal regimes. To illustrate this it is useful to idealise the heat producing layer as an infinitely thin horizon contributing all the crustal heat production.

Using this idealisation and the thermal parameters appropriate to central Australia deduced above (conductivity, crustal heat production, mantle heat flow) calculate crustal geotherms as this layer is buried at 0, 5, 10 and 15 km depth. Note how much the temperature at 40 km depth (the approximate depth of the Moho) changes as the enriched layer is moved up and down.

<div align="right">Question 2.12</div>

Integrating the steady-state heat equation*

In the steady-state ($\frac{\partial T}{\partial t} = 0$) with heat flow in the vertical direction, z, the thermal energy balance can be expressed as an ordinary differential equation:

$$0 = k \frac{d^2 T}{d z^2} + \rho H \tag{2}$$

To obtain an expresion relating temperatures and depth (as is required to calculate geotherns) we need to integrate this equation twice. As you should know, the main problem with integration lies in the evaluation of the integration constants, which are normally given by the boundary conditions that apply to the problem. For example, assuming a constant distribution of heat producing elements with depth, Eqn 2 can be integrated using the boundary conditions appropriate to the continental lithosphere: $q = -q_s$ at $z = 0$ (i.e., the heat flow at the surface of the lithosphere in the direction of increasing depth is the negative of the surface heat flow), and $T = T_s$ at $z = 0$ (ie., the temperature at the top of the lithosphere is the surface temperature).

The first integration of Eqn 2 gives:

$$0 = k \frac{dT}{d z} + \rho H z + c_1 \tag{3}$$

where c_1 is the first integration constant. At $z = 0$, Eqn 3 reduces to

$$c_1 = -k \frac{dT}{dz}\Big|_{z=0}$$

giving $c_1 = -q_s$. A further integration of Eqn 3 gives

$$0 = kT + \frac{\rho H z^2}{2} - q_s z + c_2 \qquad (4)$$

where c_2 is the second integration constant. At $z = 0$, Eqn 4 reduces to $T|_{z=0} = T_s$ giving $c_2 = -k T_s$. Adding this expression for c_2 into Eqn 4 and rearranging yields:

$$T = T_s + \frac{q_s z}{k} - \frac{\rho H z^2}{2k} \qquad (5)$$

relating the temperature at any depth, z, to the conductivity, surface temperature, T_s, surface heat flow, q_s, and heat production, H.

Mathematically-minded students should try and derive Eqn 1, and the ones that follow, from the energy equation (use the boundary condition $q = -q_m$ at $z = z_c$ for evaluating the first integration constant).

3 Transient heat flow

In the previous section we looked at problems involving steady-state heat flow in the vertical direction; that is, problems in which there is no change of temperature of any material point with time. The steady state implies that

$$\frac{\partial T}{\partial t} = 0 \qquad (6)$$

and consequently the heat equation reduces to the ordinary differential equation (ODE) :

$$0 = k \frac{\partial^2 T}{\partial z^2} + H \rho \qquad (7)$$

Since this equation is written in terms of differentials it needs further modifications to be useful to geologists who (generally-speaking) are more interested in the explicit functional relationship between temperature and depth than in differentials! The last (optional) part of practical 2 outlined the procedure (an integration) that is needed to recast (or solve) this ODE in

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terms of geologically meaningful expressions relating temperatures explicitly to depth. From the mathematical point of view the integration of ordinary differential equations is relatively simple because the constants of integration are indeed just constants and are relatively easily deduced from the boundary or initial conditions that prescribe the problem (such as the temperature at the surface of the Earth).

In actively deforming regions of the Earth's crust, or in regions close to magmatic intrusions or with laterally variable thermal parameters, the steady state approximation is not valid, and we need to consider the partial differential form of the heat equation

$$\frac{\partial T}{\partial t} = \kappa \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + \frac{H}{C_p} \tag{8}$$

Clearly, this partial differential equation (PDE) is structurally more complex than the ODE above (Eqn 7), because it is expressed in terms of differentials of temperature with regard to both time and space (in fact PDE's are defined as differential equations in which the differential is taken with regard to more than one variable). The methods entailed in solving partial differential equations are weird and wonderful (mainly because the integration requires finding new functions rather than constants). Because of this complexity we will not try here to solve Eqn 8 directly but rather we will analyze a solution that has already been derived for a geologically meaningful situation - the cooling of dyke like intrusion.

The cooling dyke

The cooling of a dyke and the thermal evolution of the surrounding country rock into which it is intruded provides a useful model for contact metamorphism. In this situation, we are only interested in horizontal component of heat flow in the direction perpindicular to the dyke wall, and we can neglect the last term in Eqn 8 (which expresses the contribution due to internal radioactive heat sources and which is insignificant compared to the heat flowing from the hot dyke):

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial x^2} \tag{9}$$

The solution to Eqn 9 appropriate to the cooling dyke problem is given by the following expression

$$T = 0.5V \left(\text{Erf} \left(\frac{a - x}{2(\kappa t)^{.5}} \right) + \text{Erf} \left(\frac{a + x}{2(\kappa t)^{.5}} \right) \right) \qquad (10)$$

where a is the half width of the dyke, V is the temperature difference between the dyke and the surrounding country rock, x is the distance from the centre of the dyke and Erf is the error function.

Solutions to Eqn 10 for a 400 metre wide dyke intruded at 500°C above the surrounding rock are shown in Figure 7.

The area under the curve defining the temperature field across a cooling dyke is controlled by a fundamental conservation principle! What is this principle and how does it effect the size of the area?   **Question 3.1**

Using the data in the lower diagram in Figure 7, draw temperature-time curves appropriate to points located 20 m, 100 m, 400 m and 700 m beyond the dyke margin (draw all the T-t paths on the same diagram).   **Question 3.2**

Using the data in the upper diagram in Figure 7, draw schematic curves showing

- **the temperature gradient (grad T or $\partial T/\partial x$) from the centre of the dyke to 500 m for the times 10, 100, 1000, 10000 after intrusion.**

- **the gradient in the temperature gradient (grad(grad T) or $\partial^2 T/\partial x^2$) from the centre of the dyke to 500 m for the times 10, 100, 1000, 10000 after intrusion).**

Question 3.3

Place the second figure immediately underneath the first - with the same horizontal scaling! What feature controls the locus of the maximum rates of heating, and how does this relate to the various terms in Eqn 9.

What attribute of the temperature field controls the magnitude of the term $\partial^2 T/\partial x^2$ **Question 3.4**
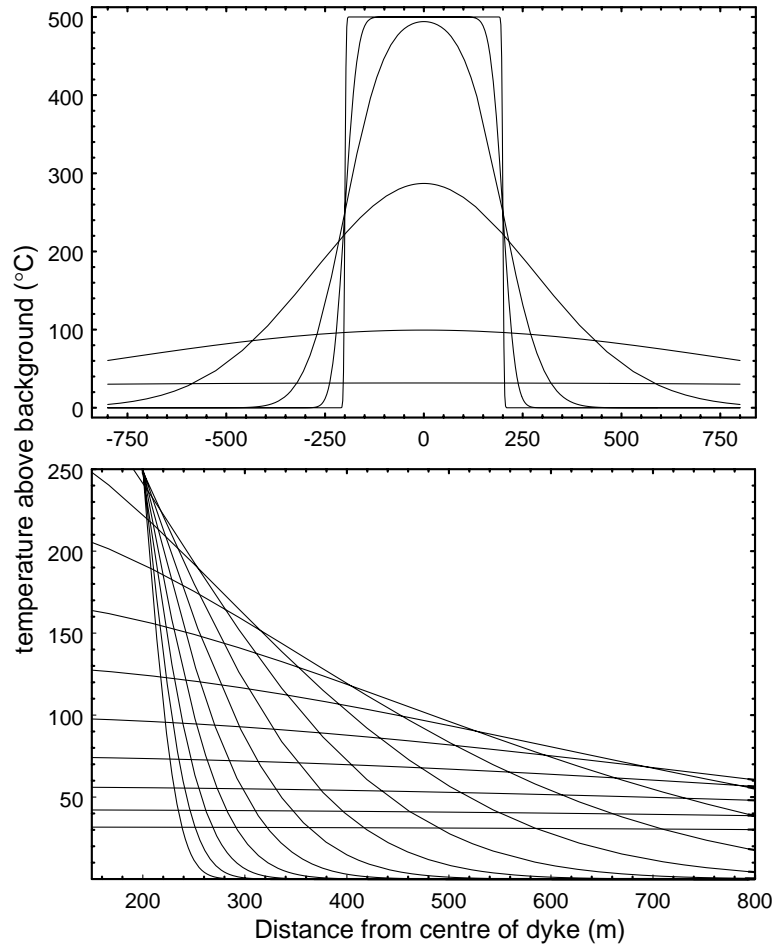
Figure 7: Illustration of the thermal effects of a dyke on the surrounding country rock. The upper diagram shows the temperature field across a 400 m wide dyke intruded at a temperature of 500 ^{o}C above the surrounding country rock, at times 0.1, 10, 100, 1000, 10000 and 100000 years after intrusion. The lower diagram shows a subset of the region shown in the upper diagram with the temperature field shown for times 10, 18, 32, 56, 100, 178, 316, 562, 1000, 1778, 3162, 5623, 10000, 17782, 31622, 56234 and 100000 years after intrusion.

4 Projections

Compatibility diagrams provide a useful way of visualizing the chemical relationships amongst mineral phases in metamorphic rocks, and are particularly useful in helping us to visualize what reactions may take place between the minerals. The compatibility diagram that we will use in this practical is the AFM diagram, which is particularly useful for illustrating the *chemographic relationships* in pelitic schists.

The aim of the first exercise is to provide an introduction to the use of compatibility diagrams and projections. Firstly, plot quartz, andalusite and almandine on a FeO-SiO_2-Al_2O_3 triangle. Project andalusite and almandine from quartz onto the FeO-Al_2O_3 binary. Confirm that this is the same as simply ignoring SiO_2 in the mineral formula. Why is this? Next, Add Fe-staurolite and Fe-cordierite to the FeO-Al_2O_3 binary. What additional component do we need to project from in order to make the projection of staurolite strictly valid?   Question 3.1

Consider the FeO-Al_2O_3 binary as one side of the ternary compatibility diagram $KAlO_2$-FeO-Al_2O_3 projected from quartz and H_2O. Add muscovite and biotite. Now, graphically project biotite from muscovite onto FeO-Al_2O_3. Try to algebraically project biotite from muscovite onto FeO-Al_2O_3 (ie. write the composition of biotite in temrs of muscovite and FeO-Al_2O_3, and then normalise the coordinates of FeO and Al_2O_3). Confirm that these two procedures give the same solution!   Question 3.2

The plane MgO-FeO-Al_2O_3 projected from muscovite, quartz and H_2O is Thompson's AFM projection, appropriate to pelitic rocks. The projection is a simple extension of the FeO-Al_2O_3 binary because all the minerals plotted so far, bar andalusite, involve the $MgFe_{-1}$ exchange vector. Plot the tie triangles for the following assemblages on an AFM diagram:
st - bt - cd ($X_{Fe,st} = 0.8$, $X_{Fe,bt} = 0.47$, $X_{Fe,cd} = 0.31$)
st - cd - and ($X_{Fe,st} = 0.75$, $X_{Fe,cd} = 0.25$)   Question 3.3

the mineral Compositions are :

- muscovite (mu): $K_2Al_6Si_6O_{20}(OH)_4$

- garnet (gt): $(Mg, Fe)_3Al_2Si_3O_{12}$

- staurolite (st): $(Mg, Fe)_2Al_9Si_{3.75}O_{22}(OH)_2$

- biotite (bt): $K_2(Mg, Fe)_6Al_2Si_6O_{20}(OH)_4$

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- cordierite (cd): $(Mg,Fe)_2Al_4Si_5O_{18}$

Note that the problem of plotting coordinates of minerals on a ternary diagram (or in a 2-space) often seems to be unduly diffficult. I do not understand why this should be. However, it may be worth while considering what happens in the more familiar spaces of lower dimension:

- A 0-dimensional compositional space (or, more simply a 0-space) can consist of only one component. All phases in a 0-space must therefore have the same composition (that necessarily corresponds to the compositon of the one system defining component).

- A 1-space is defined by only two components. All phases must consist of some proportion of the two components and therefore plot on a line. The position on the line is simply obtained by determining the relative proportions of the two components in the phase.

- We can in fact easily extend this to any space: an n-space is defined by n+1 components. All phases plot in an n-dimensional hyperspace with the coordinates given by the proportions of the n+1 components. (Admittedly, this is pretty difficult to imagine for n ¿ 3).

Of course, the use of projections allows our phases to have both positive and negative values of the components. Thus in a 1-space the line between the components represents the region of compositional space where phases have positive proportions of both components. The other parts of the line represent the regions where one of the components has a negative value (this merely sates the obvious, ie, the composition of a 1-space can be represented by an inifinitely long line). Similarly, the composition of a 2-space can be represented by an infinite plane in which there three fundamentally different regions; the region where all components have positive values (for instance, inside the triangle on an AFM diagram), regions where one component has a negative value, eg. biotite projection on the AFM diagram) and, finally, regions where two components have negative values. Of course, it is not possible to have a phase in 2-space in which the proportions of all three components are negative since the proportions of all components in a phase must sum to unity.

The AFM diagram entails a projection onto a compositional space defined by two independently variable components (and therefore, unlike the AKF and ACF diagrams that you might read about in various texts, constitutes a valid compatibility diagram). The composition of phases to be shown on the AFM diagram must be projected from a complex multi-dimensional composition space (usually at least 5-dimensions) onto a two dimensional space defined by the ternary system Al_2O_3-FeO-MgO, abbreviated to AFM.

To achieve this projection mineral compositions are projected from the saturating phases (phases that are always present in pelites such as quartz, muscovite or K-feldspar, H_2O) onto the AFM plane. To obtain the projecting co-ordinates of any phase we wish to display on the AFM diagram we simply need to write a balanced reaction between the phase to be plotted, the compositions of the three apices of the plot (namely, Al_2O_3, FeO and MgO) and the saturating phases that are always assumed to be present (namely, muscovite, quartz and H_2O fluid). The plotting co-ordinates are then given by the three coefficients for Al_2O_3, FeO and MgO normalised to unity.

A simplified procedure, illustrated below with an example involving biotite, achieves the same result as writing the full balanced reaction with *much less effort*. In the simplified procedure (for assemblages containing muscovite) the projection co-ordinates are given by the proportions of (Al_2O_3 - 3 x K_2O), FeO and MgO in the phase to be projected, normalised to unity (in assemblages with K-feldspar instead of muscovite the projection co-ordinates are given by the proportions of (Al_2O_3 - K_2O), FeO and MgO normalised to unity).

Biotite has the limiting composition $K_2(Fe, Mg)_6Al_2Si_6O_{20}(OH)_4$. So that ($Al_2O_3$-3 x K_2O) defines a negative number. This is no problem as it simply reflects the fact that when projecting from muscovite, biotite plots further away from the A-apex of the AFM diagram than the line connecting the F- and M-apices. For instance, for a biotite with an Mg:Fe ratio of 2:1, the projection co-ordinates are calculated as follows:

(Al_2O_3 - 3 x K_2O) = 1 - 3 = -2
FeO = 2
MgO = 4
(Al_2O_3 - 3 x K_2O) + Fe0 + MgO = 4
co-ordinates of A = -0.5
co-ordinates of F = 0.5
co-ordinates of M = 1.0

Using a projection from muscovite, quartz and H_2O, plot the following phases on an AFM diagram assuming the Mg : Fe ratio as specified:

- garnet = 1:4

- staurolite = 2:7

- kyanite -

- cordierite = 3:2

- chlorite = 2:1

- biotite = 1:1

Question 3.4

5 Compatibility diagrams, continuous and discontinuous reactions

Consider a system involving the following phases : biotite (bt), muscovite (mu), quartz (q), garnet (gt), staurolite (st), andalusite (and), cordierte (crd), and H_2O (v), with compositions restricted to KFMASH. How many discontinuous (or univariant) reactions occur in between this sytems. Use an AFM compatibility diagram to determine the reactants and products for each reaction.

Question 3.1

The typical prograde sequence in the Mount Lofty ranges involves the following succession of mineral assemblages in pelitic schists. For each zone, assemblages are listed in order of increasing bulk rock Mg/Fe ratio. All assemblages are with muscovite and quartz.

Question 3.2

- biotite zone: **bt-chl**;

Question 3.3

- garnet zone: *bt-gt*, *bt-gt-chl*, **bt-chl**;

- staurolite zone: **bt-gt, bt-gt-st, bt-st bt-st-chl, bt-chl**;

- andalusite zone (1): **bt-gt, bt-gt-st, bt-st, bt-st-and, bt-and,** *bt-and-crd*, *bt-crd*;

- andalusite zone (2): **bt-gt, bt-gt-and, bt-and,** *bt-and-crd*, *bt-crd*;

- sillimanite zone: **bt-gt, bt-gt-sill, bt-sill,** *bt-sill-crd*, *bt-crd*;

Note, that since naturally occurring pelites have relatively restricted range of bulk rock X_{Fe} (typically 0.9-0.7) it is not possible to deduce the complete sequence of AFM assemblages on the basis of field studies alone. Those assemblages listed in italics are those inferred to be possible, but which have not yet been documented (to my knowledge) from the Mount Lofty Ranges.

Assuming the following order of Fe-enrichment:

$$X_{Fe,gt} > X_{Fe,st} > X_{Fe,bt} > X_{Fe,chl} > X_{Fe,crd}$$

(with chlorite plotting in AFM beneath the cordierite-biotite tieline) draw schematic AFM compatibility di-

Question 3.4

agrams for each of these zones. What discontinuous reactions mark the zone boundaries?

Calculated T-X diagrams projected from biotite, muscovite and quartz appropriate to low P pelites such as the Mount Lofty Ranges and other so-called " Buchan facies series" are shown in Figures 6 & 7 (after Dymoke & Sandiford,1992). Which of the two diagrams is more
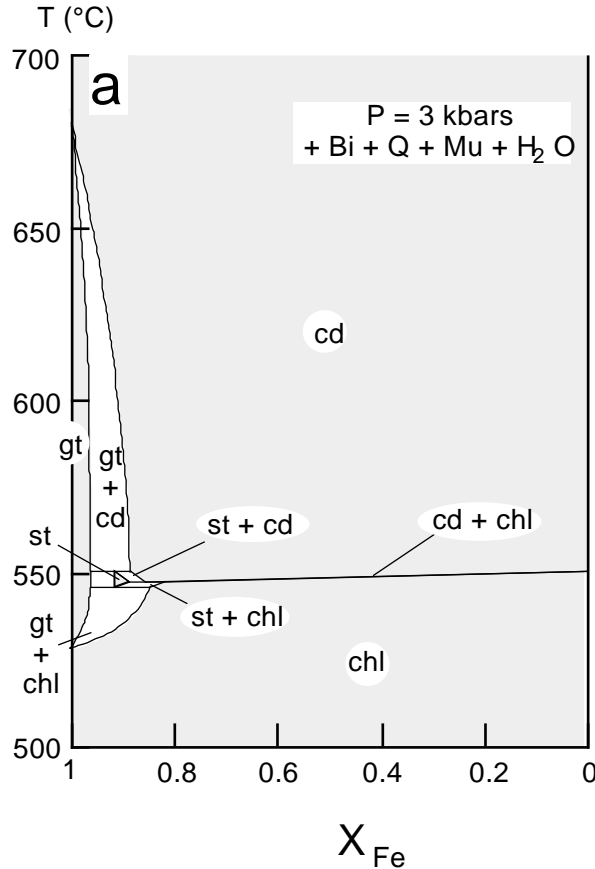

Figure 8:

appropriate to the sequence described above. For the appropriate diagram describe the sequence of changing mineralogy, including explicit reference to both continuous (divariant) and discontinuous (univariant) reactions for rocks with X_{Fe} corresponding to 0.9, 0.8 and 0.65, respectively. Show schematically on AFM diagrams how the tie triangles for divariant assemblages (i.e. those containing three phases on the AFM diagram) swing between each discontinuous (univariant)

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reaction with increasing temperature.

Is the appropriate diagram for the sequence described above also appropriate to the mineral association also found in Sample 185-389. What change in environmental parameters may give rise to the texture in Sample 185-389.
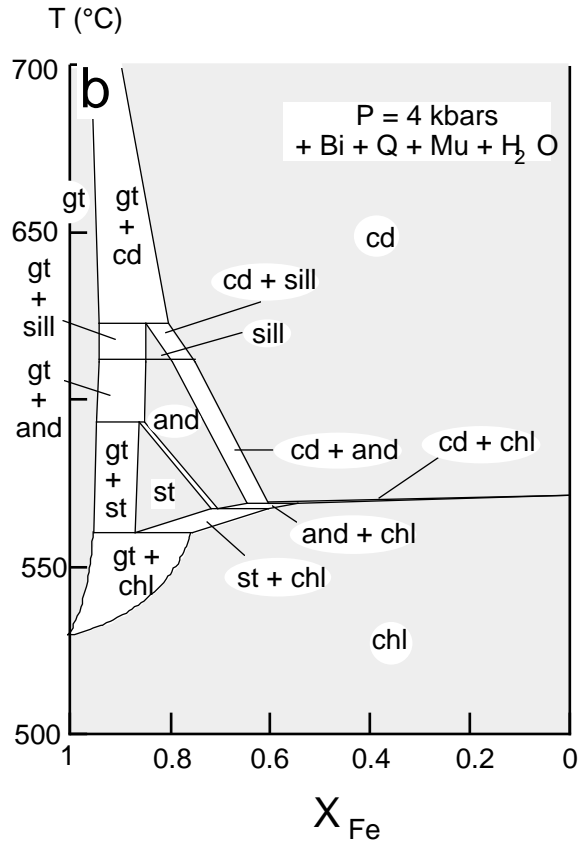
T (°C)

Figure 9:

6 Schreinemakers analysis

In practical 3 we explored some of the reaction relationships between the phases garnet, staurolite, andalustie, cordierite, biotite, muscovite, quartz and an aqueous vapour. In this six-component system (KFMASH) we have 8-phases; it is, therefore, an (n+2)-system and must include one invariant point. Considering biotite, muscovite, quartz and vapour to be in excess, there are four univariant reactions of interest (as determined in the last Practical 3). The aim of this exercise is to construct a $P - T$ grid for the KFMASH **Question 4.1** system above (plus chlorite) including the appropriate compatibility diagrams for each of the $P - T$ fields.

The principal problem in Schreinemakers analysis is to determine which, if indeed any, of the possible topologies may be stable. This can often be done if we have some additional knowledge of the assemblages which occur in nature, or if there are some appropriate experimental results available. In the above example staurolite occurs only at high pressures.

Consider KFMASH with the 8 phases used in the previous exercise plus chlorite, which now makes an (n+3) - system and so must include more than one invariant point. A topology which seems to account for the natural assemblages in biotite-, muscovite-, quartz-, vapour-bearing rocks is one involving the stable invariant points [garnet] and [chlorite] (note that [chlorite]. Construct the appropriate topology, again with **Question 4.2** all reactions with positive dT/dP.

It is probable that [garnet], [chlorite], and [aluminosilicate] all occur within the andalusite field in KFMASH, with [garnet] at about the temperature of the triple point. Draw the schematic grid with the alumi- **Question 4.2** nosilicate phase diagram superposed on it.

In the practicals and lectures we have constructed a number of different diagrams; $G - T$ diagrams, $G - X$ diagrams, compatibility diagrams, $T - X$ diagrams ($P - X$ diagrams will look just like $T - X$ diagrams) and now $P - T$ diagrams. I have tried to emphasize the relationship between these different types of diagrams; for example, every compatibility diagram represents a projection along the G-axis from a corresponding

$G-X$ diagram. for example you should be aware of the connection netween $G-X$ and T-X diagrams. One of the most difficult aspects in $P-T$ diagrams is to see how they relate to $T-X$ or $P-X$ diagrams, of the type used, for example, in Practical **3**. Can you see how these two types of diagrams relate (if you can you are really getting somewhere).

In order to determine precisely the slopes of the reactions and $T-P$ co-ordinates, we will need to incorporate thermodynamic data. However, it can generally be assumed that dehydration reactions are temperature sensitive (vapour in the high T side) with positive dP/dT.