4 The strength of the lithosphere

Constitutive equations specify the relations between stress and strain or the time derivatives of strain and thus can be used in conjunction with the equations of motion to relate stresses and displacements. A number of idealized classes of constitutive relations are recognized. The principal material behaviours are *elastic*, *plastic* and *viscous*. It should be noted that the constitutive relations appropriate to these behaviours are idealized and many materials show more complicated stress-strain relations. For example, a special class of behaviour of relevance to geology is *visco-elasticity* (and visco-plasticity).

4.1 A rheological primer

In simple, isotropic, elastic materials stress and strain are linearly related. An important aspect of elasticity is that all strain is recovered on relaxation of the stress. Plastic material shows an elastic response up to a critical stress, termed the yield stress, at which the material fails. For perfectly plastic behaviour the material cannot support stresses greater than the yield stress. In viscous materials stresses and strain rates are related. Thus no deformation is recovered with the relaxation of stresses. The general form of the constitutive relation for viscous materials is :

$$\tau = k\,\dot{\gamma}^{\frac{1}{n}} \tag{4.1}$$

where k and n are constants, and $\dot{\gamma}$ is the shear strain rate. For $n = 1$ the relationship between stress and strain rate is linear and the material is said to be Newtonian with the viscosity equal to k. Materials with $n > 1$ are termed *power-law* or *shear thinning* fluids. For high values of n the behaviour of the material may approximate plasticity in as much as a dramatic change in the rate of deformation occurs with the relatively small increases in shear stress.

Visco-elastic materials show time dependent behaviour which is like elastic materials at short time-scales and like viscous materials at longer time-scales as illustrated in the response to the application of a step-like shear strain in Figure 4.1 and in a creep test in Figure 4.2.

4.2 Background to lithospheric rheology

The distribution of seismicity in the ocean basins suggests that the ocean lithosphere deforms primarily by the rigid body translation and rotation of large "plates"; as we shall see these motions are sustained by the push of the ocean ridges and the pull of the subducting slabs. The lack of internal deformation of the ocean lithosphere reflects its strength; it must be strong enough to sustain the stresses arising from these fundamental driving forces (that is, the ocean lithosphere acts as a stress guide). The behaviour of the ocean lithosphere contrasts with many parts of the continents wherein intense, diffuse, internal deformation is manifested by the wide distribution of seismic events.

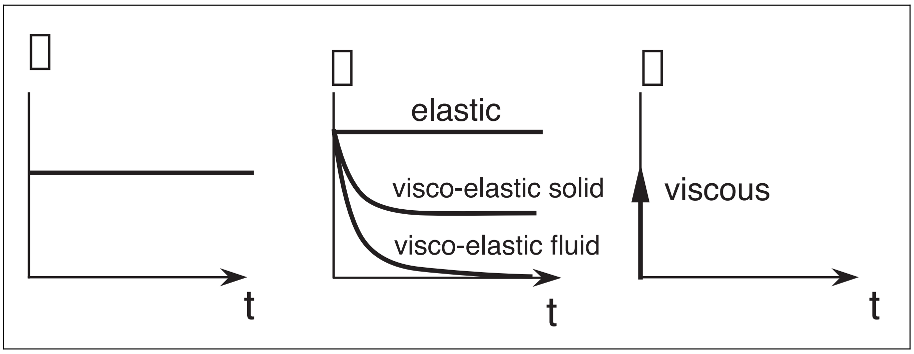

Figure 4.1 Elastic, viscous and visco-elastic response to a step shear strain applied at time t=0.

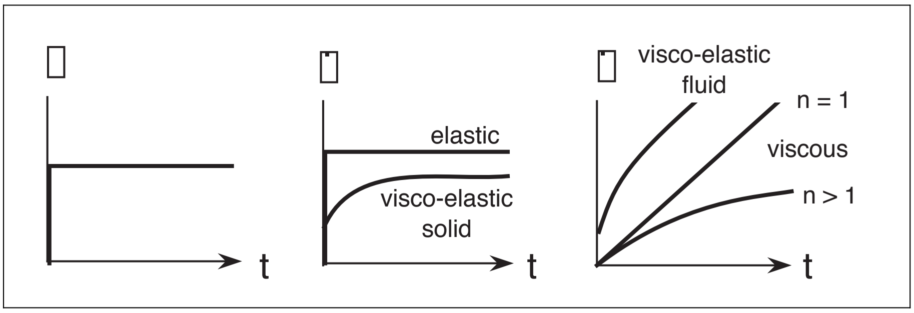

Figure 4.2 Elastic, viscous and visco-elastic response to a creep test.

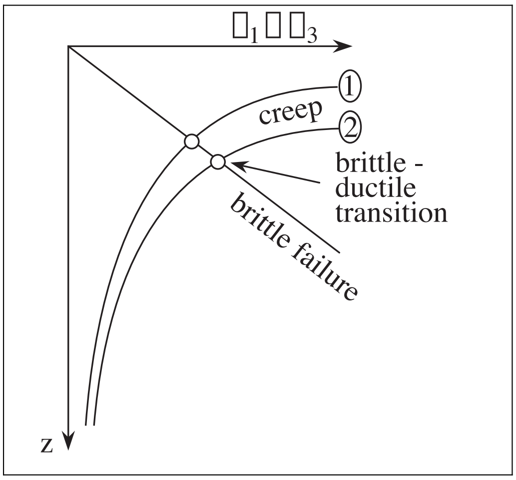

Figure 4.3 The stress needed to deform the lithosphere and the failure mode depend upon the depth, the strain rate, the thermal structure as well as the mineralogical composition of the deforming rocks. At shallow depths (low confining pressures and low temperatures) failure occurs in the brittle mode, at deeper levels (high confining pressures and high temperatures) deformation occurs in a ductile fashion. The strength of rocks is strongly dependent on the temperature and the strain rate. Curve 1 is appropriate to low strain rates or high geotherms, while curve 2 is appropriate to lower geotherms or higher strain rates. The brittle-ductile transition can be viewed as the depth at which failure mode switches, and coincides approximately with the base of the seismogenic zone.

The existence of diffuse deformation within the continental interiors implies that the continental lithosphere is, in general, considerably weaker than the oceanic lithosphere (or that the magnitude of the stresses sustaining deformation of the continents is larger than for oceans). In addition, differences in the behaviour of the oceanic and continental lithosphere are governed by differences in their buoyancy. Old oceanic lithosphere is negatively buoyant allowing eventual subduction and the effective recycling of the oceanic lithosphere in the convective mantle; one consequence is that there is no oceanic lithosphere older than about 200 Ma on the surface of the modern Earth. In contrast the continental lithosphere is always positively buoyant and cannot be subducted to any significant extent; consequently we have fragments of the continental lithosphere dating back to about 3.9 Ga.

In later chapters we consider the deformation of the continental lithosphere in both compression (which results in the development of mountain belts) and in tension (which results in the development of rifts and the attendant sedimentary basins and flanking uplifted margins). We will not be so much concerned with the internal geometry of deformation (which is very much the realm of structural geology) but the mechanical and thermal consequences of the deformation on the lithospheric scale. Our aim is to develop an understanding of the controls on some simple and regular, yet, as we shall see, fundamentally important features of continental orogenic belts, for example, the *first-order* controls on elevation. In order to understand the behaviour of the continents during deformation we begin with some simple notions concerning the strength or rheology of the continental lithosphere.

4.3 A model lithosphere

The above discussion implies that the lithosphere has finite strength! That is rocks are able to sustain a finite deviatoric stress (or stress difference) by elastic deformation, all of which is recoverable on the relaxation of the stress. At stress in excess of the critical deviatoric stress rocks will undergo a permanent deformation by processes such as brittle fracture, dislocation creep, pressure solution, the actual deformation mechanism depending on the temperature, strain rate and composition of the rock. The finite elastic strength of the lithosphere implies the general rheological response is plastic, and since the response of the lithosphere can be shown in many cases to be time dependent[11] the lithosphere exhibits a complex form of visco-plasticity. The complete description of the rheological behaviour of such a lithosphere represents a formidable challenge. However it is possible to achieve the general form of a visco-plastic response by specifying a lithospheric rheology governed by a combination of two simple failure mechanisms, as described below.

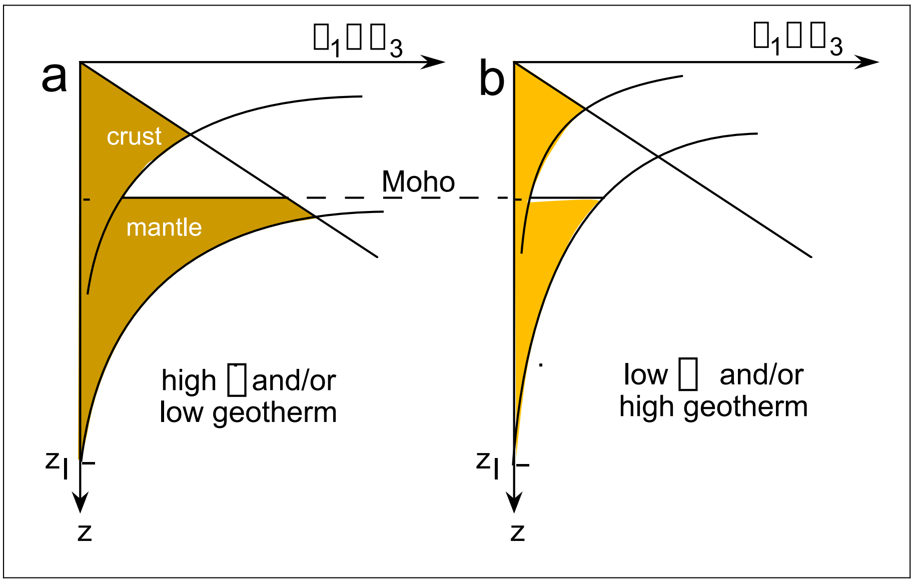

Figure 4.4 The strength of the lithosphere is given by the area under the τ vs depth curves. The strength is a function primarily of the vertical compositional structure of the lithosphere, the geotherm and the strain rate. (a) is appropriate to a low geotherm or high strain rate, i.e. strong lithosphere, where the strongest part of the lithosphere is the brittle upper mantle. (b) is appropriate to a high geotherm or low strain rate (i.e., a weaker lithosphere) where there is no brittle mantle. Note that the proportion of the total strength of the lithosphere concentrated in the crust (especially near the brittle-ductile transition) increases with increasing geotherm.

[11] For example, in flexural basins

Active seismicity in the continents is, by and large, restricted to depths less than about 15 kms. Since seismic energy represents the release of elastic energy at failure along a discrete fault plane, the lower limit of seismicity can be thought of as the brittle-ductile transition. The constitutive law describing failure in the brittle mode by a frictional sliding mechanism is frequently called Byrelee's law:

$$\tau = c_o + \mu\,\sigma_n \tag{4.2}$$

where c_o is the cohesion, τ is the shear stress required for failure ($\tau = (\sigma_1 - \sigma_3)/2$), σ_n is the normal stress on the failure plane and μ is the coefficient of friction. Since the normal stress acting across the failure plane increases with depth the stress needed to cause brittle failure also increases with depth (as a linear function of depth for the case where the density is constant with depth).

The constitutive laws describing deformation in the ductile regime have a power-law form:

$$\frac{(\sigma_1 - \sigma_3)}{G} = \left(\frac{\dot{\gamma}}{A}\right)^{\frac{1}{n}} exp\left(\frac{(Q + P\,V^*)}{R\,T\,n}\right) \tag{4.3}$$

where G is the shear modulus (units of Pa), A is a material constant with units s^{-1}, $\dot{\gamma}$ is the shear strain rate, Q is a material constant known as the activation energy of units J mol^{-1}, V^* is the activation volume in m^3 mol^{-3}, and n is a dimensionless material constant known as the power law exponent. The exponential term in Eqn ??eq:creep1 expresses the inverse exponential dependence of strength on temperature which obeys an Arhenius relationship. The $PV*$ term is usually small compared to $Q$and thus Equation ?? can be approximated by:

$$\frac{(\sigma_1 - \sigma_3)}{G} = \left(\frac{\dot{\gamma}}{A}\right)^{\frac{1}{n}} exp\left(\frac{Q}{R\,T\,n}\right) \tag{4.4}$$

Eqns ?? and ?? show that the strength of rocks to creep decreases exponentially with increasing temperature and increases with the strain rate. Thus for a given composition, strength must decrease with depth.

In terms of this rheology the brittle-ductile transition can therefore be understood as the depth where the stress required for failure is equal for both ductile creep and brittle failure (Figure 4.3). The material constants for creep vary significantly with the mineralogical makeup of the rock. For example, quartz-rich rocks are much weaker than olivine-bearing rocks. Thus the strength of the lithosphere can be determined only for a specific compositional structure (as well as thermal structure). To a first approximation we can consider that the continental lithosphere comprises two layers: a quartz-rich crustal layer and an olivine-rich mantle layer. For any arbitrary geotherm the strength of such a hypothetical lithosphere depends only on the material constants appropriate to quartz and olivine (Figure ??fig:rfour). Permanent deformation of the whole lithosphere at given shear strain rate, $\dot{\gamma}$, will only occur when the force applied to the lithosphere exceeds the strength of the lithosphere, F_l, given by (Figure 4.5b):

$$F_l = \int_0^{z_l} (\sigma_1 - \sigma_3)\, dz \tag{4.5}$$

For high strain rates, low geotherms or low applied forces much of the stress in the lithosphere will be supported at least in part elastically (Figure 4.5a). Because of the dependence on strain rate the behaviour of the lithosphere is time dependent. On short time scales the lithosphere is able to support forces in an elastic fashion; on longer timescales it will relieve the applied forces by viscous, permanent deformation. The rheology of the lithosphere may therefore be considered as visco-plastic.

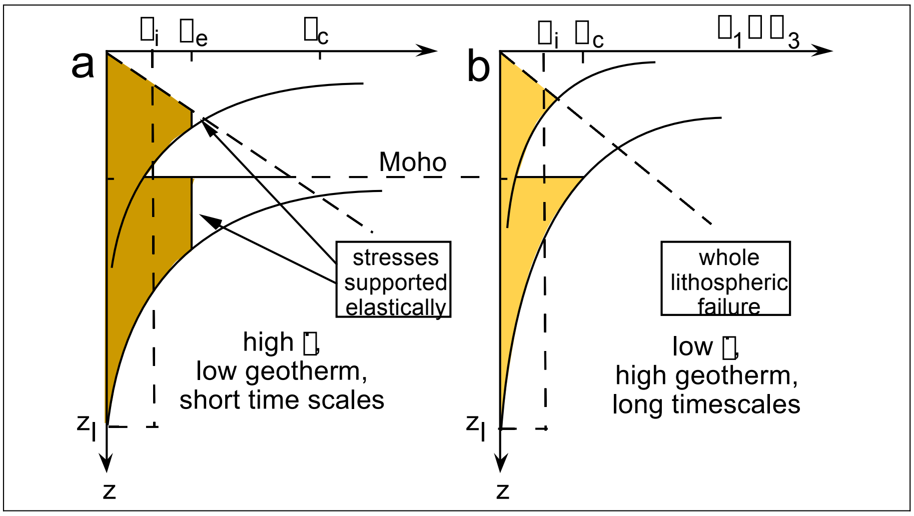

Figure 4.5 The distribution of stress within the lithosphere subject to a horizontal force (in compression or tension) depends on the magnitude of the force, the strain rate and the geotherm (for a given composition). An applied force, F_d, will give instantaneously a homogeneous stress distribution, σ_i, throughout the lithosphere such that $\sigma_i\, z_l = F_d$ In those regions where σ_i exceeds the critical stress difference for permanent deformation the stresses will be relaxed, with resultant amplification of the stress in the stronger parts of the lithosphere. Permanent deformation of the whole lithosphere can only occur when the whole lithosphere fails (b), that is when the stress concentration everywhere exceeds the stress difference required for permanent deformation. For lower applied forces the stresses will be supported at least in part elastically (a).

4.4 Uncertainties

The simple model for lithospheric rheology outlined in the previous section is highly uncertain for a number of reasons, and consequently must be used with some caution. Firstly, the lithosphere comprises polymineralic aggregates and the extent to which the whole of the crust can be described by *quartz-failure* and the mantle by *olivine-failure* is extremely dubious. The strength-depth curve for more realistic compositional models of the lithosphere is likely to be much more complex than shown in models shown in Figure 4.4 & 4.5. Secondly, the material constants are determined in laboratory experiments carried out at strain rates appropriate to human activity (10^{-5}–$10^{-8}\mathrm{s}^{-1}$), as opposed to geological strain rates (10^{-13}–$10^{-16}\mathrm{s}^{-1}$). This is particularly important because only small changes in the value of the material constants have a large effect on the calculated strength. Finally, the model excludes a number of deformation mechanisms, such as pressure solution which are known to be operative at least under some conditions.

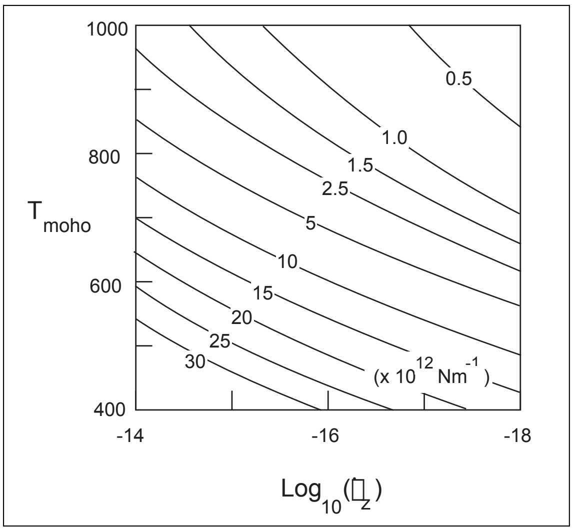

Figure 4.6 Illustration of the dependence of lithospheric strength calculated using our simple rheological model on thermal state of the lithosphere, as reflected by the Moho temperature.

Such uncertainties render futile the calculation of the absolute strength of the lithosphere using this simplistic model. For example, a 10% uncertainty in the activation energy for creep corresponds to an uncertainty in the strength of the lithosphere of about a factor of 2. However, much more significance can be attached to estimates of the changes in strength accompanying changes in the physical state of the lithosphere using this model, because at least in this case uncertainties in the material constants cancel. For example, a change in the thermal regime of the lithosphere corresponding to a change in the Moho temperature of 100°C causes a change in the strength of about a factor of 2 (Figure 4.6).