Appendix A

A.1 The equations of motion

Consider a small rectangular parallelepiped aligned in a cartesian coordinate system, x, y and z, with sides of length δ_x, δ_y and δ_z respectively, as shown in Figure A.1. Remembering that force equals stress by area we begin by decomposing the surfaces forces into stresses acting on each of the faces.

Assume that at the point, P, centred within the parallelpiped the normal stresses in the directions x, y and z are given by σ_{xx}, σ_{yy} and σ_{zz}. The shear stress in the direction normal to x and parallel to y is given by τ_{xy} and parallel to z by τ_{xz}. Similarly the shear stresses normal to y are given by τ_{yx} and τ_{yz}, and normal to z by τ_{zx} and τ_{zy}. Then the forces acting in the direction x on the faces BB'CC' and AA'DD' are, respectively:

$$\left\{\sigma_{xx} + \frac{1}{2}\frac{\partial \sigma_{xx}}{\partial x}\delta_x\right\}\delta_y\delta_z$$

and

$$-\left\{\sigma_{xx} - \frac{1}{2}\frac{\partial \sigma_{xx}}{\partial x}\delta_x\right\}\delta_y\delta_z \tag{A.1}$$

where the negative sign is due the fact that stresses are treated as positive in tension and negative in compression. Across the faces A'B'C'D' and ABCD the forces are

$$\left\{\tau_{xy} + \frac{1}{2}\frac{\partial \tau_{yx}}{\partial y}\delta_y\right\}\delta_x\delta_z$$

and

$$-\left\{\tau_{xy} - \frac{1}{2}\frac{\partial \tau_{yx}}{\partial y}\delta_y\right\}\delta_x\delta_z \tag{A.2}$$

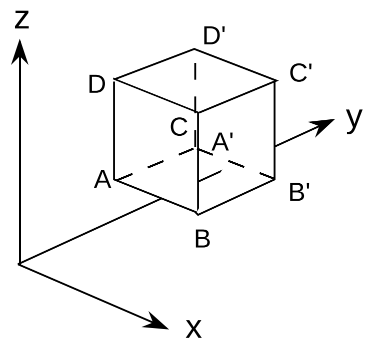

Figure A.1: Co-ordinate system used to derive the equations of motion

Across the faces DCC'D' and ABB'A' the forces are

$$\left\{\tau_{xz} + \frac{1}{2}\frac{\partial \tau_{zx}}{\partial z}\delta_z\right\}\delta_x\delta_y$$

and

$$-\left\{\tau_{yz} - \frac{1}{2}\frac{\partial \tau_{zx}}{\partial z}\delta_z\right\}\delta_x\delta_y \tag{A.3}$$

The body force acting on the volume with density, ρ, in the direction x is given by

$$\rho X\delta_x\delta_y\delta_z \tag{A.4}$$

The total force in the x direction is then

$$\left\{\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + \rho X\right\}\delta_x\delta_y\delta_z \tag{A.5}$$

If the component of displacement of point P in the x direction is u then Newton's second law gives

$$\rho\frac{\partial^2 u}{\partial^2 t} = \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + \rho X \tag{A.6}$$

Similar results obtain for the y and z directions. In tectonic settings accelerations can be regarded as neglible, and the only body force is gravity which acts in the vertical direction, taken to be z, and using the convention for summation over repeated indices, the equations of motion can be reduced to:

$$0 = \frac{\partial \tau_{i,j}}{\partial x_i} + a_i\rho g \tag{A.7}$$

where a_i is the unit vector $(0, 0, 1)$. Note that in the convention for indices adopted in Eqn A.7 the co-ordinates x, y and z are given by x_1, x_2 and x_3, respectively, while $\sigma_{ii} = \tau_{ii}$. Equation A.7 is general and can be applied to many problems related to tectonic phenomona. However, since it is couched in terms of the components of the stress tensor it must be rendered useful through combination with *constitutive equations* relating stresses to displacements.

A.2 Calculation of ridge-push force

In order to solve Eqn. 7.7 we need to formulate the density distribution appropriate to Figure 7.2a. The appropriate density distribution is (Figure 7.2b):

$$\begin{aligned} \rho_z &= \rho_m, \quad t = 0, \quad 0 < z \\ \rho_z &= \rho_w, \quad t = t_1, \quad 0 < z < w \\ \rho_z &= \rho_m \left[1 + \alpha\ (T_m - T_z)\right], \quad t = t_1, \quad w < z < w + z_l \end{aligned} \tag{A.8}$$

where ρ_m is the density of mantle at T_m, the temperature of the asthenosphere, ρ_w is the density of water, α is the volumetric coefficient of thermal expansion of peridotite. The density distributions defined by Eqn 7.7 give the following variation $(\sigma_{zz})_z$:

$$\begin{aligned} (\sigma_{zz})_z &= \rho_m\, g\, z, \quad t = 0, \quad 0 < z \\ (\sigma_{zz})_z &= \rho_w\, g\, z, \quad t = t_1, \quad 0 < z < w \\ (\sigma_{zz})_z &= \rho_w\, g\, w + g \int_w^{w+z_l} \rho_z dz, \quad t = t_1, \quad w < z < w + z_l \end{aligned} \tag{A.9}$$

F_1 and F_2 are given by:

$$F_1 = \int_0^{w+z_l} (\sigma_{zz})_z\ dz = \frac{\rho_m\, g\ (w + z_l)^2}{2} \tag{A.10}$$

$$F_2 = \int_0^{w} (\sigma_{zz})_z\ dz = \frac{\rho_w\, g\, w^2}{2} \tag{A.11}$$

Since in the lithosphere the density is itself a function of depth the third term, F_3, in Eqn 7.7 is given by:

$$F_3 = \int_w^{w+z_l} (\sigma_{zz})_z\ dz \tag{A.12}$$

Assuming that the lithospheric geotherm at t_1 is linear in depth, the temperature at the surface of the lithosphere $T_s = 0^o$C, and α is independent of temperature then:

$$T_z = T_m \frac{z}{z_l}, \; \rho_z = \rho_m \left(1 + \alpha\, T_m \left(1 - \frac{z}{z_l}\right)\right)$$

then

$$g \int_w^{w+z_l} \rho_z \, dz = g\, z_l\, \rho_m + \frac{g\, z_l\, \rho_m\, \alpha\, T_m}{2} = g\, z_l\, \rho_m \left(1 + \frac{\alpha\, T_m}{2}\right)$$

and

$$F_3 = \rho_w\, g\, w\, z_l + \frac{g\, z_l^2\, \rho_m}{2} \left(1 + \frac{\alpha\, T_m}{2}\right) \tag{A.13}$$

Thus Eqn 7.7 is given by

$$F_R = \frac{\rho_m\, g\, (w + z_l)^2}{2} - \frac{\rho_w\, g\, w^2}{2} - \left(\rho_w\, g\, w\, z_l + \frac{g\, z_l^2\, \rho_m}{2} \left(1 + \frac{\alpha\, T_m}{2}\right)\right) \tag{A.14}$$

The condition of isostatic compensation at depth, $w + z_l$, requires that

$$\sigma_{zz\,(z=w+z_l,\, t=0)} = \sigma_{zz\,(z=w+z_l,\, t=1)}$$

Solving for z_l gives:

$$z_l = \frac{w\,(\rho_m - \rho_w)}{\alpha\, \rho_m\, T_m}$$